OCR
Oxford Cambridge and RSA

Wednesday 8 June 2016 – Morning

A2 GCE MATHEMATICS

4730/01 Mechanics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4730/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

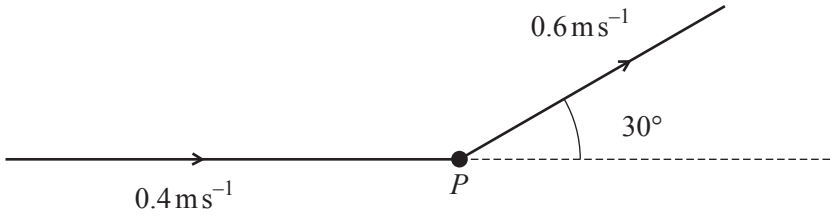
INFORMATION FOR CANDIDATES

- This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1



A particle P of mass 0.3 kg is moving with speed 0.4 m s^{-1} in a straight line on a smooth horizontal surface when it is struck by a horizontal impulse. After the impulse acts P has speed 0.6 m s^{-1} and is moving in a direction making an angle 30° with its original direction of motion (see diagram).

(i) Find the magnitude of the impulse and the angle its line of action makes with the original direction of motion of P. **[4]**

Subsequently a second impulse acts on P. After this second impulse acts, P again moves from left to right with speed 0.4 m s^{-1} in a direction parallel to its original direction of motion.

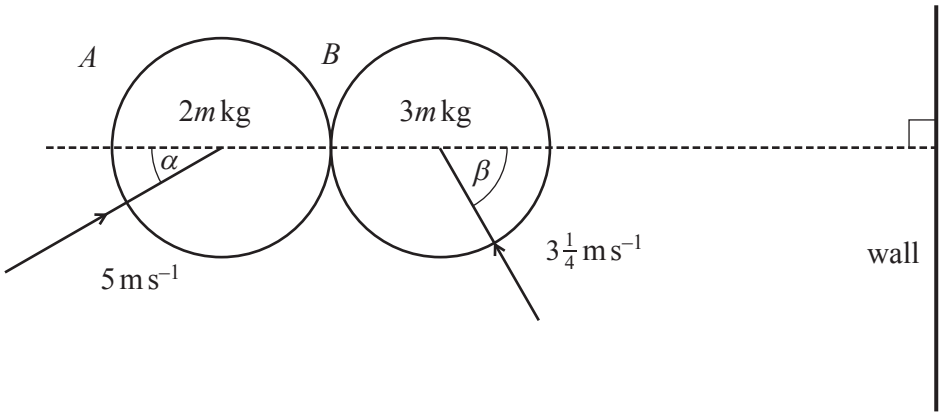
(ii) State the magnitude of the second impulse, and show the direction of the second impulse on a diagram. **[2]**

2 A particle Q of mass 0.2 kg is projected horizontally with velocity 4 m s^{-1} from a fixed point A on a smooth horizontal surface. At time t s after projection Q is x m from A and is moving away from A with velocity v m s^{-1}. There is a force of $3\cos 2t$ N acting on Q in the positive x-direction.

(i) Find an expression for the velocity of Q at time t. State the maximum and minimum values of the velocity of Q as t varies. **[4]**

(ii) Find the average velocity of Q between times $t = \pi$ and $t = \frac{3}{2}\pi$. **[4]**

3



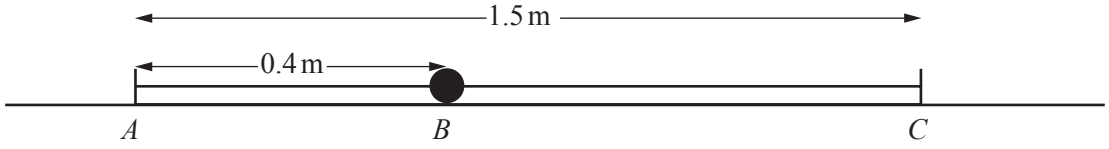
Two uniform smooth spheres A and B, of equal radius, have masses $2m$ kg and $3m$ kg respectively. The spheres are approaching each other on a horizontal surface when they collide. Before the collision A is moving with speed 5 m s^{-1} in a direction making an angle α with the line of centres, where $\cos\alpha = \frac{4}{5}$, and B is moving with speed $3\frac{1}{4}$ m s^{-1} in a direction making an angle β with the line of centres, where $\cos\beta = \frac{5}{13}$. A straight vertical wall is situated to the right of B, perpendicular to the line of centres (see diagram). The coefficient of restitution between A and B is $\frac{2}{3}$.

(i) Find the speed of A after the collision. Find also the component of the velocity of B along the line of centres after the collision. **[7]**

B subsequently hits the wall.

(ii) Explain why A and B will have a second collision if the coefficient of restitution between B and the wall is sufficiently large. Find the set of values of the coefficient of restitution for which this second collision will occur. **[3]**

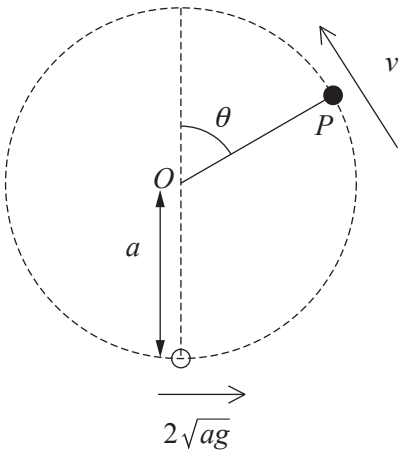
4



A and C are two fixed points, 1.5 m apart, on a smooth horizontal plane. A light elastic string of natural length 0.4 m and modulus of elasticity 20 N has one end fixed to point A and the other end fixed to a particle B. Another light elastic string of natural length 0.6 m and modulus of elasticity 15 N has one end fixed to point C and the other end fixed to the particle B. The particle is released from rest when ABC forms a straight line and $AB = 0.4$ m (see diagram).

Find the greatest kinetic energy of particle B in the subsequent motion. **[7]**

5



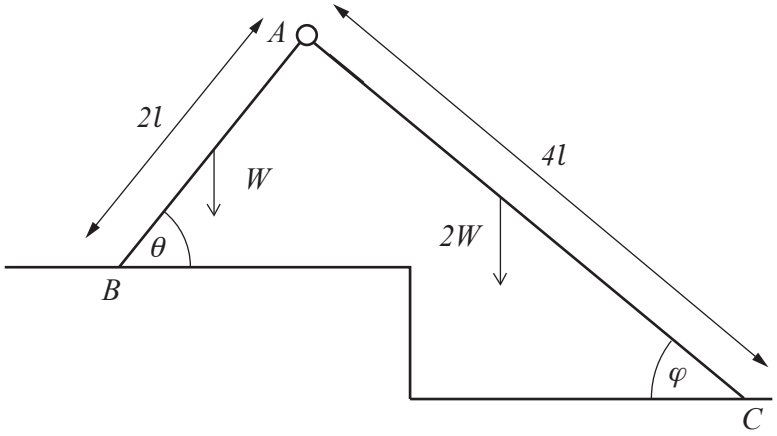
One end of a light inextensible string of length a is attached to a fixed point O. A particle P of mass m is attached to the other end of the string and hangs at rest. P is then projected horizontally from this position with speed $2\sqrt{ag}$. When the string makes an angle θ with the upward vertical P has speed v (see diagram). The tension in the string is T.

(i) Find an expression for T in terms of m, g and θ, and hence find the height of P above its initial level when the string becomes slack. **[6]**

P is now projected horizontally from the same initial position with speed U.

(ii) Find the set of values of U for which the string does not remain taut in the subsequent motion. **[5]**

6



Two uniform rods AB and AC are freely jointed at A. Rod AB is of length $2l$ and weight W; rod AC is of length $4l$ and weight $2W$. The rods rest in equilibrium in a vertical plane on two rough horizontal steps, so that AB makes an angle of θ with the horizontal, where $\sin\theta = \frac{4}{5}$, and AC makes an angle of φ with the horizontal, where $\sin\varphi = \frac{3}{5}$ (see diagram). The force of the step acting on AB at B has vertical component R and horizontal component F.

(i) By taking moments about A for the rod AB, find an equation relating W, R and F. **[3]**

(ii) Show that $R = \frac{73}{50}W$, and find the vertical component of the force acting on AC at C. **[6]**

(iii) The coefficient of friction at B is equal to that at C. Given that one of the rods is on the point of slipping, explain which rod this must be, and find the coefficient of friction. **[4]**

7 A particle P of mass m kg is attached to one end of a light elastic string of modulus of elasticity $24mg$ N and natural length 0.6 m. The other end of the string is attached to a fixed point O; the particle P rests in equilibrium at a point A with the string vertical.

(i) Show that the distance OA is 0.625 m. **[2]**

Another particle Q, of mass $3m$ kg, is released from rest from a point 0.4 m above P and falls onto P. The two particles coalesce.

(ii) Show that the combined particle initially moves with speed $2.1\,\text{m s}^{-1}$. **[3]**

(iii) Show that the combined particle initially performs simple harmonic motion, and find the centre of this motion and its amplitude. **[5]**

(iv) Find the time that elapses between Q being released from rest and the combined particle first reaching the highest point of its subsequent motion. **[7]**

END OF QUESTION PAPER

		Answer	Marks	Guidance	
1	(i)	0.3x0.4 + I_x = 0.3x0.6cos30° I_y = 0.3x0.6sin30° Magnitude of I = 0.0969 (Ns) 68.3° from original direction OR correct triangle of momentum $I^2 = 0.18^2 + 0.12^2 - 2 \times 0.18 \times 0.12 \times \cos 30°$ I = 0.096890 (Ns) 68.26° from original direction	M1 M1 A1 A1 **4** (M1) (A1) (A1) (A1)	I_x = (±) 0.035884572 I_y = (±) 0.09 CAO Accept 111.74° if clear on diagram Allow for wrong angle CAO Accept 111.74° if clear on diag.	Similar with velocities. 0.096890 68.26. Direction must be clear in words or on diagram Similar with velocities. Or slip in figure
	(ii)	Magnitude = 0.0969 N Direction opposite to (i) clearly shown and labelled or stated	B1ft B1ft **2**	Accept direction at angle 141.7° with direction of 0.6 m s^{-1} motion	If total 0 scored, allow SC1 for 38.3° or 68.3° seen
2	(i)	$3\cos 2t = 0.2\frac{dv}{dt}$ $7.5\sin 2t = v\ (+c)$ oe $v = 7.5\sin 2t + 4$ 11.5 (ms^{-1}) and −3.5 (ms^{-1})	M1* *M1 A1 A1 **4**	Use of F = ma; condone wrong / missing 0.2 and wrong sign Attempt to integrate, one side correct; condone missing c, CAO Depends on both M marks and fully correct working	
	(ii)	$x = -\frac{15}{4}\cos 2t + 4t\ (+c)$ Ave speed is their distance/$\frac{\pi}{2}$ $\left(\frac{15}{4} + 6\pi\right) - \left(-\frac{15}{4} + 4\pi\right)$ 8.77 (ms^{-1})	M1* *M1 *M1 A1 **4**	Ft if (i) has sin or cos term {$x(\frac{3}{2}\pi) - x(\pi)$} found; CAO Accept $\frac{15}{\pi} + 4$	No need to find c (3.75) $\left(\frac{15}{2} + 6\pi\right) - (+4\pi)$

		Answer	Marks	Guidance	
3	(i)	C of M $2m \times 5\cos\alpha - 3m \times 3\frac{1}{4}\cos\beta = 2ma + 3mb$	M1*	allow sign and number slips.	a and b are vels of A and B to right
		$2m$x4 – $3m$x1.25 = $2ma$ + $3mb$	A1	($2a + 3b = 4.25$)	
		Newton's experimental law	M1*	Or equivalent; allow sign slips	
		$b - a = -\frac{2}{3}(-1.25 - 4)$	A1	($b - a = 3.5$)	Consistent
		Attempt to solve simultaneous equations	*M1		
		b = 2.25 (m s^{-1})	A1	CAO	−2.25 if b defined to left
		(a = − 1.25 so) speed of A = 3.25 (m s^{-1})	A1 **7**	CAO	
	(ii)	A and B both have same component of velocity perp to l o c	B1		
		After collision with wall B must move faster than A	B1	May be implied	
		Coeff of restitution > 5/9 (accept > 0.5 recurring, or 0.556)	B1 **3**	Do not allow ≥	Ignore $e \leq 1$, etc

		Answer	Marks	Guidance	
4		Initial energy = $\frac{15\times0.5^2}{2\times0.6}$	B1	3.125 (J) or $\frac{25}{8}$	
		$\frac{20x}{0.4}=\frac{15(0.5-x)}{0.6}$	M1A1	x measured from B; other points possible	
		$x=\frac{1}{6}$ (m)	A1	OR $\frac{17}{30}$ from A, or $\frac{11}{60}$ from midpt	Or 1/3 + 0.6 from C
		$\frac{20\times\left(\frac{1}{6}\right)^2}{2\times0.4}$ or $\frac{15\times\left(0.5-\frac{1}{6}\right)^2}{2\times0.6}$ seen	B1	Allow their $\frac{1}{6}$	Not their $\frac{17}{30}$ or $\frac{11}{60}$
		KE = $3.125-\frac{20\times\left(\frac{1}{6}\right)^2}{2\times0.4}-\frac{15\times\left(0.5-\frac{1}{6}\right)^2}{2\times0.6}$	M1	Allow their $\frac{1}{6}$	$\frac{25}{8}-\frac{25}{36}-\frac{25}{18}$
		= 1.04167 (J)	A1 **7**	= (3.125) – 0.69444 – 1.3888 Or $\frac{37.5}{36}$ or $\frac{25}{24}$	$\frac{75}{72}$
		OR KE = $[3.125]-\frac{20x^2}{2\times0.4}-\frac{15(0.5-x)^2}{2\times0.6}$	(M1)	All terms present	CTS $-37.5\left(\frac{1}{6}-x\right)^2+\frac{37.5}{36}$
		Differentiate OR complete the square	(M1)	Allow slips	leads to max KE = $\frac{37.5}{36}$
		Max KE when $x=\frac{1}{6}$	(A1)	B1 & B1M1A1 as above	
		OR stationary when $3.125=\frac{20x^2}{2\times0.4}+\frac{15(0.5-x)^2}{2\times0.6}$	(M1A1)		
		x = 0 or $\frac{1}{3}$ so max EPE when $x=\frac{1}{6}$	(A1)	B1 & B1M1A1 as above	
		Or by SHM, $m\ddot{x}=\frac{15(0.5-x)}{0.6}-\frac{20x}{0.4}$	(M1A1)	Or other x	
		$m\ddot{x}=-75\left(x-\frac{1}{6}\right)$	(A1)		
		Amplitude, $a=\frac{1}{6}$	(B1)		
		$\omega^2=\frac{75}{m}$	(B1)		
		Max KE = $\frac{1}{2}m\left(\frac{1}{6}\right)^2\times\frac{75}{m}$	(M1)		
		$\frac{25}{24}$ J (or 1.04)	(A1)		

		Answer	Marks	Guidance	
5	(i)	By energy $\frac{1}{2}m(4ag) = \frac{1}{2}mv^2 + mga(1+\cos\theta)$ Use of $F = ma$ $T + mg\cos\theta = \frac{mv^2}{a}$ $T = 2mg - 3mg\cos\theta$ Slack when $\cos\theta = \frac{2}{3}$ Height is $\frac{5}{3}a$	M1 A1 M1 A1 M1 A1 **6**	Need correct 3 terms; allow wrong sign, missing/extra g, missing m / a; sin for cos correct Need 3 terms and g, allow sign slip, sin for cos Ft if their T has right form	$v^2 = 2ag - 2ag\cos\theta$
	(ii)	If $\theta > \pi/2$, $\frac{1}{2}mU^2 > mga$ $U > \sqrt{(2ag)}$ For no complete revolutions $\frac{1}{2}mU^2 < \frac{1}{2}mu^2 + 2mga$ and $mg = m\frac{u^2}{a}$ $U < \sqrt{(5ag)}$ OR Use $\frac{1}{2}mU^2 = \frac{1}{2}mv^2 + mga(1+\cos\theta)$ and $T + mg\cos\theta = \frac{mv^2}{a}$ To get $T = m\frac{U^2}{a} - 2mg - 3mg\cos\theta$ oe When T = 0, $U^2 = 2ag + 3ag\cos\theta$ $(\theta = 0)$ gives $U < \sqrt{(5ag)}$ $(\theta = \frac{\pi}{2})$ gives $U > \sqrt{(2ag)}$	M1 A1 M1 M1 A1 **5** (M1) (A1) (M1) (A1) (A1)	Allow '=' for all M marks Allow ≥ Allow wrong sign Allow wrong sign Allow ≤ Allow ≤,< Allow ≤ Allow ≥	u is vel at top $\sqrt{2ag} < U < \sqrt{5ag}$

Answer			Marks	Guidance	
6	**(i)**	Moments about *A* for *AB*	M1	Allow if sign errors, sinθ / cosθ present; dim correct	Allow sin/cos confusion.
		$Wl\frac{3}{5} + F2l\frac{4}{5} = R2l\frac{3}{5}$	A1	Allow sign errors	
		$3W + 8F = 6R$ oe	A1 **3**	CAO	
6	**(ii)**	Moments about *A* for *AC*	M1	Allow if sign errors, sinθ / cosθ present	*Q* is normal reaction at *C* Allow sin/cos confusion.
		$2W2l\frac{4}{5} + G4l\frac{3}{5} = Q4l\frac{4}{5}$	A1	Allow sign errors ($16W + 12G = 16Q$ oe)	*G* is friction force at *C*
		$F = G$	B1		
		Solve with eqn from (i) and $R + Q = 3W$	M1		dep moments equation
		$R = \frac{73}{50}W$	A1	AG	Accept $1.46W$
		$Q = \frac{77}{50}W$	A1 **6**	If < 4 marks, sc B1 for $R + Q = W + 2W$	Or $1.54W$
		Or moments about C for whole system	(M1)	Allow if sign errors, sinθ / cosθ present. Need 4 terms.	Allow sin/cos confusion.
		$2W \times 2l\frac{4}{5} + W\left(4l\frac{4}{5} + l\frac{3}{5}\right) - R\left(4l\frac{4}{5} + 2l\frac{3}{5}\right) = F\left(4l\frac{3}{5} - 2l\frac{4}{5}\right)$	(A1)	Allow sign errors	
		$(35W - 22R = 4F)$	(A1)	Correct	
		Solve with eqn from (i)	(M1)	Final A1A1 as main scheme	dep moments equation
	(iii)	Attempt to find $F\ (= G) = \frac{18}{25}W$ or $F\ (= G) = \frac{36}{73}R$	M1	Or 0.72 W Ft their normal forces if at least M1 in (i) and (ii)	Or use (i) with $F=\mu R$ and $R = \frac{73}{50}W$ M1A1
		AB	B1	Can be implied	$584\mu W = 438W - 150W$
		Coeff of friction $= \frac{18}{25}W \div \frac{73}{50}W$	M1	Or 0.72W/1.46W May see $\frac{18}{25}W \div \frac{77}{50}W$ at $C\left(\frac{36}{77}\right)$	Solve A1; B1 as scheme
		$\frac{36}{73}$ or 0.493(15)	A1 **4**	CAO	

		Answer	Marks	Guidance	
7	(i)	In equilibrium, $mg = \frac{24mge}{0.6}$ (e = 0.025 m); OP = 0.625 m	M1 A1 **2**	Working essential, with 24mg and 0.6 used AG	
	(ii)	For Q falling 0.4m $v^2 = 2g \times 0.4$ v = 2.8 (m s^{-1}) For combined particle 3mx2.8 = 4mV V = 2.1 (m s^{-1})	 M1 A1 A1 **3**	 Or $\frac{1}{2}mv^2 = mg \times 0.4$ Accept $\sqrt{(0.8g)}$ Valid method shown – no slips **AG**	
	(iii)	When a distance x below O $4mg - \frac{24mg(x-0.6)}{0.6} = 4ma$ $-10g(x-0.7) = \ddot{x}$ (SHM about) a point 0.7 m below O $2.1^2 = 10g(a^2 - 0.075^2)$ amplitude = 0.225 (m)	 M1 A1 B1 M1 A1 **5**	Or $4mg - \frac{24mg(x+0.1)}{0.6} = 4m\ddot{x}$ Accept a for $\ddot{x}$; $-10gx = \ddot{x}$ Allow their ω; any x	Allow sign error, m for 4m $4mg - \frac{24mgx}{0.6} = 4m\ddot{x}$ $-10g(x-0.1) = \ddot{x}$ Use of $v = a\omega$ gains M1
	(iv)	$0.4 = \frac{1}{2}9.8t^2$; or 2.8 = 9.8t $0.075 = 0.225\sin(\sqrt{98}t)$ $\frac{1}{2}T = \frac{\pi}{\sqrt{98}}$ $0.1 = 0.225\sin(\sqrt{98}t)$ $v = (0.225\sqrt{98}\cos(\sqrt{98} \times 0.046523)$ (= 1.995) or $v^2 = 98(0.225^2 - 0.1^2)$ $0 = 1.99 - 9.8t$ Time = 0.888 s	M1 M1 M1 M1 M1 M1 A1 **7**	Falling dist 0.4 m: 0.285714 s Travel 0.075m to centre: 0.0343287 s Half period: 0.31734878 s Travel 0.1m above centre: 0.046523 s Speed when string becomes slack 1.995307 m s^{-1} Travel to highest point: 0.203602 s 0.8875175 **CAO**	Must be their 2.8 Allow their values; allow cos Allow for whole or part period Allow their values; allow cos Allow their values; allow sin Allow their values Dep M6